Announcements

1) Practice Problems for HW 2 up.

On CTools under "Assignments"

NOT for a grade.

2) Turn in 2 problems from HW 2

Recall:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided limit exists.

Reminder: $f'(a)$ is equal to the slope of the tangent line to the graph of f at $x = a$.

Leibniz Notation

Sometimes we write

$$\frac{df}{dx}$$ for the derivative

of f with respect to

x. Means the same

as f'.

Example 1: $f(x) = 1$

find $\dfrac{df}{dx}$.

$$\frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{1 - 1}{h}$$

$$= \lim_{h \to 0} \frac{0}{h} = \boxed{0}$$

Example 2: $f(x) = x$

Find f'.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = \boxed{1}$$

Example 3: Find

$f'(x)$ if

$$f(x) = x^2 + \frac{17}{x^3-1} + \sqrt{2-x}$$

No.

I want a better way to find $f'(x)$.

Todays class = finding a better way

Shortcuts for Differentiation

1) Constant Rule

If c is a constant and f is differentiable at $x = a$,

$$\boxed{\frac{d}{dx}(c \cdot f) = c \frac{df}{dx}}$$

Differentiation pulls out constants.

Application: $f(x) = 5$, $f'(x) = 0$

2) Sum Rule

Suppose $f'(x)$ and $g'(x)$ both exist at $x=a$.

Then

$$\boxed{(f+g)'(a) = f'(a) + g'(a)}$$

Comment: Both the sum and constant rules follow directly from limit laws

Back to example 3

$$f(x) = x^2 + \frac{17}{x^3-1} + \sqrt{2-x}$$

The sum rule tells you that

$$\frac{df}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}\left(\frac{17}{x^3-1}\right) + \frac{d}{dx}\left(\sqrt{2-x}\right)$$

and by the constant rule,

$$\frac{df}{dx} = \frac{d}{dx}(x^2) + 17\frac{d}{dx}\left(\frac{1}{x^3-1}\right) + \frac{d}{dx}\left(\sqrt{2-x}\right)$$

3) Difference Rule

Suppose $f'(x)$ and $g'(x)$ exist at $x=a$.

Then

$$(f-g)'(a) = f'(a) - g'(a)$$

Follows from the sum and constant rules

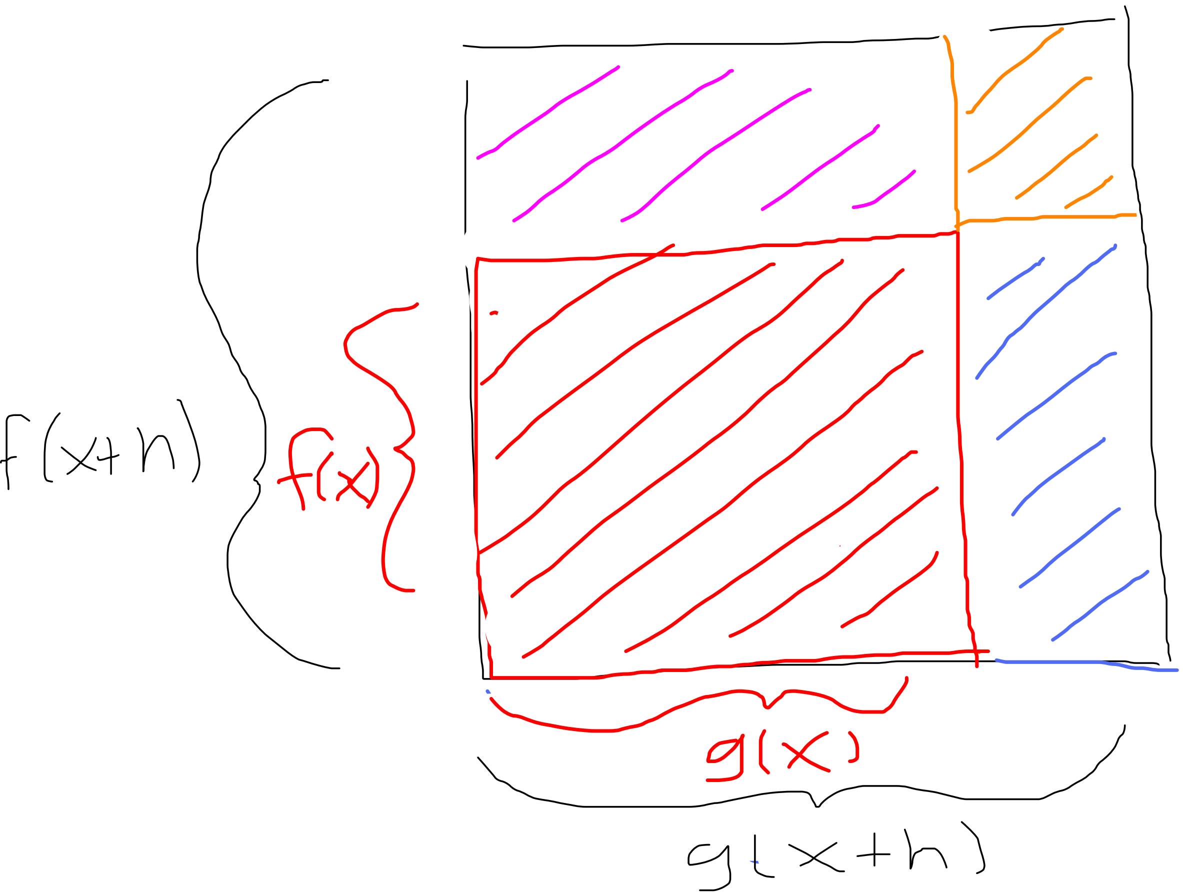
4) Product Rule!

Let $f'(x)$ and $g'(x)$ exist at $x=a$. Then

$$\boxed{(f\cdot g)'(a) = f'(a)g(a) + g'(a)f(a)}$$

$$(f\cdot g)'(a) = \lim_{h\to 0} \frac{\boxed{f(x+h)g(x+h) - f(x)g(x)}}{h}$$

Picture



(Area of black retangle) −

(Area of red rectangle

$$= \boxed{f(x+h)\,g(x+h) - f(x)g(x)}$$

It also equals

Area of blue rectangle

+ Area of orange rectangle

+ Area of purple rectangle

$$f(x)\left(g(x+h)-g(x)\right)$$

$$+\left(f(x+h)-f(x)\right)\left(g(x+h)-g(x)\right)$$

$$+\left(f(x+h)-f(x)\right)g(x)$$

As $h \to 0$, the area of the orange rectangle disappears. For small h,

$$f(x+h)g(x+h) - f(x)g(x)$$
$$= \big(g(x+h) - g(x)\big) f(x)$$
$$+ \big(f(x+h) - f(x)\big) g(x).$$

To get $(f \cdot g)'$, divide by h. Take limit as $h \to 0$.

Then

$$\lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \lim_{h \to 0} \frac{(g(x+h) - g(x))f(x) + (f(x+h) - f(x))g(x)}{h}$$

$$= \lim_{h \to 0} f(x)\left(\frac{g(x+h) - g(x)}{h}\right)$$

$$+ \lim_{h \to 0} g(x)\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= f(x) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$+ g(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \boxed{f(x)g'(x) + g(x)f'(x)}$$

Application of Product Rule

[Know!]

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(x \cdot x)$$

$$= x\frac{dx}{dx} + x\frac{dx}{dx} \quad \text{(product rule)}$$

$$= x + x = \boxed{2x}$$

$$\frac{d}{dx}(x^3) = \frac{d}{dx}(x \cdot x^2)$$

$$= x \frac{d}{dx}(x^2) + x^2 \frac{d}{dx} x$$

(product rule)

$$= x \cdot 2x + x^2 \cdot 1 = \boxed{3x^2}$$

<u>Generalize</u> Let r be any real number

$$\frac{d}{dx}(x^r) = r x^{r-1}$$

5) Power Rule

If r is any real number,

$$\boxed{\frac{d}{dx}\left(x^r\right) = r x^{r-1}}$$

6) Chain Rule!

Suppose $f'(x)$ and $g'(x)$ exist at $x = a$,

$$\boxed{(f \circ g)'(a) = f'(g(a)) \cdot g'(a)}$$

Example. Find $\frac{d}{dx}\left(\sqrt{2-x}\right)$

$$\sqrt{2-x} = (2-x)^{1/2}$$

$$= (f \circ g)(x) \quad \text{where}$$

$$g(x) = 2 - x$$

$$f(x) = \sqrt{x} = x^{1/2}$$

Use Chain Rule.

$$\frac{d}{dx}\left(\sqrt{2-x}\right) = \frac{d}{dx}\left(f \circ g(x)\right)$$

$$= f'(g(x))\, g'(x)$$

$$g(x) = 2 - x$$
$$f(x) = x^{1/2}$$

$$g'(x) = -1$$
$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

So $f'(g(x))\,g'(x)$

$$= \frac{1}{2}(2-x)^{-1/2} \cdot (-1)$$
$$= \boxed{-\frac{1}{2}(2-x)^{-1/2}}$$

7) Quotient Rule

Let $f'(x)$ and $g'(x)$

exist at $x=a$. Then

$$\boxed{\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}}$$

Shortcut mnemonic:

$$d\left(\frac{hi}{lo}\right) = \frac{lo(d\,hi) - hi(d\,lo)}{(lo)^2}$$

d = derivative

Example:

$$\frac{d}{dx}\left(\frac{1}{x^3-1}\right)$$

Use quotient rule...

$$\frac{d}{dx}\left(\frac{1}{x^3-1}\right) = \frac{(x^3-1)\cdot 0 - 1\cdot 3x^2}{(x^3-1)^2}$$

$$= \boxed{\frac{-3x^2}{(x^3-1)^2}}$$

Back to example 3

$$f(x) = x^2 + \frac{17}{x^3-1} + \sqrt{2-x}$$

The sum rule tells you that

$$\frac{df}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}\left(\frac{17}{x^3-1}\right) + \frac{d}{dx}\left(\sqrt{2-x}\right)$$

and by the constant rule,

$$\frac{df}{dx} = \frac{d}{dx}(x^2) + 17\frac{d}{dx}\left(\frac{1}{x^3-1}\right) + \frac{d}{dx}\left(\sqrt{2-x}\right)$$

$$= \boxed{2x + 17\left(\frac{-3x^2}{(x^3-1)^2}\right) + -\frac{1}{2}(2-x)^{-1/2}}$$